¨T minus 3, 2, 1¨. Most of us see the final seconds before launch, but a countdown for a typical space shuttle mission begins about 43 hours before launch. The shuttle goes through a number of inspections and systems testing. At T – 6 hours, the shuttle begins fueling, and this process takes nearly three hours.
Learning Goals
In this lesson, you will:

- Evaluate numerical expressions with addition, subtraction, multiplication, and division.
- Evaluate numerical expressions involving exponents and parentheses.
- Justify the order of operations used to simplify numerical expressions.

Key Terms
- conventions
- numerical expression
- evaluate
- operations
- parentheses
- order of operations

Have you ever wondered why in some states cars can turn right on red? Or why in England and Ireland cars drive on the left side of the road? Why are all books written in English printed horizontally from left to right and from front to back? Did you know that traditional Chinese, Japanese, and Korean texts are written vertically from right to left? Rules like these, which are called conventions, are usually developed over time and are followed so that everyone knows what to do. Many conventions deal with the order in which something is done. For example, you usually eat your salad before your meal, and the meal before your dessert. What other conventions do you know? Do you know of any mathematical conventions?
Problem 1  There’s a Reason for the Rules

In mathematics, people follow rules to clarify the order of how to solve problems. Part of these rules include numerical expressions and strategies for calculating values. A numerical expression is a mathematical phrase containing numbers. To evaluate a numerical expression means to calculate an expression to get a single value. Many times, numerical expressions have operations that tell you what to do with each value. The operations in an expression are addition, subtraction, multiplication, and division. For example, consider the numerical expression $8 - 7 + 4 - 3$.

1. What does Jose’s solution tell you about how to add and subtract in a numerical expression?
When you need to add and subtract in a numerical expression, you must perform the operations from left to right.

2. Consider the numerical expression $6 + 4 \cdot 3$.
   What operations are represented in this expression?

3. The numerical expression $6 + 4 \cdot 3$ was evaluated in two different ways, resulting in different values.

<table>
<thead>
<tr>
<th>Solution A</th>
<th>Solution B</th>
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</thead>
<tbody>
<tr>
<td>$6 + 4 \cdot 3$</td>
<td>$6 + 4 \cdot 3$</td>
</tr>
<tr>
<td>$= 10 \cdot 3$</td>
<td>$= 6 + 12$</td>
</tr>
<tr>
<td>$= 30$</td>
<td>$= 18$</td>
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</table>

   a. Describe the strategy used in Solution A. What operation was performed first?

   b. Describe the strategy used in Solution B. What operation was performed first?

As with adding and subtracting in a numerical expression, there is a certain order to perform the operations when using addition and multiplication. To determine which solution is correct, think about what the expression really means. Remember, multiplication is repeated addition.

c. Rewrite $6 + 4 \cdot 3$ using only one operation.

d. Evaluate your new expression.

e. Is Solution A or Solution B correct? Cross out the incorrect solution. Explain what you discovered by evaluating your new expression.
4. Rewrite each numerical expression to represent only addition, and then evaluate each expression.

   a. \(2 + 8 \cdot 3\)  
   b. \(4 + 2 \cdot 5 + 3 \cdot 2\)

5. Write a rule that states the order in which you should perform addition and multiplication when evaluating a numerical expression.

6. Consider the numerical expression \(8 \cdot 2 \div 4 \cdot 3\). What operations are represented in this expression?

- Deshawn’s Solution:
  \[
  8 \cdot 2 \div 4 \cdot 3 = 16 \div 4 = 4 \cdot 3 = 12
  \]
- Miko’s Solution:
  \[
  8 \cdot 2 \div 4 \cdot 3 = \frac{16}{4} = \frac{4}{3} = 1 \frac{1}{3}
  \]

7. What does Deshawn’s solution tell you about how to multiply and divide in a numerical expression?
Just like addition and subtraction, when you have multiplication and division in a numerical expression, you must multiply and divide from left to right.

8. Consider the numerical expression $8 + 12 \div 4$.
   What operations are represented in this expression?

9. The numerical expression $8 + 12 \div 4$ was evaluated in two different ways, resulting in different values.

<table>
<thead>
<tr>
<th>Solution A</th>
<th>Solution B</th>
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<tbody>
<tr>
<td>$8 + 12 \div 4$</td>
<td>$8 + 12 \div 4$</td>
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<tr>
<td>$= 20 \div 4$</td>
<td>$= 8 + 3$</td>
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<td>$= 5$</td>
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</table>

   a. Describe the strategy used in Solution A. What operation was performed first?

   b. Describe the strategy used in Solution B. What operation was performed first?

To determine which solution is correct, think about what the expression really means. Remember, division is repeated subtraction.

c. Explain what $12 \div 4$ means as repeated subtraction.

d. Is Solution A or Solution B correct? Cross out the incorrect solution.

10. Write a rule that states the order in which you should perform addition and division when evaluating a numerical expression.
11. Evaluate each numerical expression.
   a. $12 - 8 \div 2 = \quad$ b. $3 \cdot 2 - 6 \div 3 =$
   c. $12 \cdot 5 - 8 = \quad$ d. $40 - 28 + 2 \cdot 5 =$
   e. $12 \div 4 \div 2 = \quad$ f. $28 \div 7 \cdot 2 =$

**Problem 2** What about Exponents and Parentheses?

1. Consider the numerical expression $9 + 2^3$.
   a. What does $2^3$ mean? What operation is being used?
   b. Evaluate the expression. Use the rules you wrote to help you calculate your answer. Then, explain the order in which you evaluated the expression.

2. Consider the numerical expression $2 \cdot 5^2$.

   **Miguel’s Solution**
   
   $2 \cdot 5^2$
   
   $2 \cdot 5 = 10$
   
   $10^2 = 100$

   **Doug’s Solution**
   
   $2 \cdot 5^2$
   
   $5^2 = 25$
   
   $2 \cdot 25 = 50$
3. What does Miguel's solution tell you about how to solve a numerical expression with both multiplication and exponents?

Previously, you learned that it did not matter which order you multiplied factors. However, when there are exponents and multiplication in a numerical expression, perform the exponent operation first, and then multiply.

4. Evaluate each numerical expression.
   a. $4 \cdot 7^2 =$
   b. $12 + 8^2 =$
   c. $3^2 - 6 \div 3 =$
   d. $12 + 25 \div 5^2 =$
   e. $10 \div 2 - 3 + 2^4 =$
   f. $12^2 - 48 \div 2 =$
   g. $28 \div 2^2 - 36 \div 3^2 =$
   h. $168 \div 2^3 + 3^3 - 20$

Parentheses are symbols used to group numbers and operations, and are used to change the normal order in which you perform operations.

For example, you know that $6 + 4 \cdot 3 = 18$ because you multiply first, and then add. What if you wanted to add first, and then multiply?

5. If parentheses were added to create the new expression $(6 + 4) \cdot 3$, would the solution change? If so, what is the new value of the expression? Explain your reasoning.
6. Consider the numerical expression $(3 + 5)^2$.

This numerical expression was evaluated in two different ways, resulting in different values.

<table>
<thead>
<tr>
<th>Solution A</th>
<th>Solution B</th>
</tr>
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<tbody>
<tr>
<td>$(3 + 5)^2$</td>
<td>$(3 + 5)^2$</td>
</tr>
<tr>
<td>$= 8^2$</td>
<td>$= 9 + 25$</td>
</tr>
<tr>
<td>$= 64$</td>
<td>$= 34$</td>
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</tbody>
</table>

Determine which solution is correct. State the reasons why one solution is correct and the error that was made in the other solution. Cross out the incorrect solution.

7. Let’s consider another numerical expression containing parentheses $3 \cdot (7 - 2)$.

<table>
<thead>
<tr>
<th>Solution A</th>
<th>Solution B</th>
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</thead>
<tbody>
<tr>
<td>$3 \cdot (7 - 2)$</td>
<td>$3 \cdot (7 - 2)$</td>
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<tr>
<td>$= 21 - 2$</td>
<td>$= 3(5)$</td>
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<td>$= 19$</td>
<td>$= 15$</td>
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</table>

Determine which solution is correct. State the reasons why one solution is correct and the error that was made in the other solution. Cross out the incorrect solution.
Parentheses group numbers and operations together. Parentheses change the order of how you should perform operations. Any operations in parentheses must be performed first before any other operations in a numerical expression.

8. Evaluate each numerical expression.
   a. \((3 + 5)^2\)
   
   b. \(12 + (25 \div 5)^2\)
   
   c. \(10 \div (5 - 3) + 2^2\)
   
   d. \((12^2 - 48) \div 2\)
   
   e. \((28 \div (2^2 + 3)) + 3^2\)
   
   f. \(((5 + 2 \cdot 2)^2 \div 3) - 20\)
There is an order of operations, a particular order in which operations are performed when evaluating any numerical expression. The Order of Operations is a set of rules that ensures the same result every time an expression is evaluated.

**Order of Operations Rules**

1. Evaluate expressions inside parentheses or grouping symbols such as ( ) or [ ].
2. Evaluate exponents.
3. Multiply and divide from left to right.
4. Add and subtract from left to right.

Keep in mind that multiplication and division are of equal importance and evaluated in order from left to right, as well as addition and subtraction.

We should use “Please Excuse My Dear Aunt Sally” to remember Parentheses, Exponents, Multiplication, Division, Addition, and Subtraction, right?

The mnemonic may help you remember the order. The important thing is to understand why the order of operations works.

I like “Pink Elephants Must Dance Around Snakes” better. Is that OK?
1. Each numerical expression has been evaluated correctly and incorrectly. First, state how the order of operations rules were used correctly to evaluate the expression, and then determine the error that was made in the second calculation.

a. \[4 + 3^2\]  
   \[= 4 + 9\]  
   \[= 13\]  

\[4 + 3^2\]  
\[= 7^2\]  
\[= 49\]

b. \[10 ÷ 4 + 1\]  
   \[= \frac{10}{4} + 1\]  
   \[= \frac{5}{2} + 1\]  
   \[= \frac{7}{2}\]

\[10 ÷ 4 + 1\]  
\[= 10 ÷ 5\]  
\[= 2\]

c. \[6 + 15 ÷ 3\]  
   \[= 6 + 5\]  
   \[= 11\]

\[6 + 15 ÷ 3\]  
\[= 21 ÷ 3\]  
\[= 7\]

d. \[(2 + 6)^2\]  
   \[= (8)^2\]  
   \[= 64\]

\[(2 + 6)^2\]  
\[= 4 + 36\]  
\[= 40\]
e. \[2(10 - 1) - 3 \cdot 2\]
\[= 2(9) - 3 \cdot 2\]
\[= 18 - 6\]
\[= 12\]

f. \[3(4 + 2)\]
\[= 3(6)\]
\[= 18\]

Be prepared to share your solutions and methods.
You've probably gone to a park before. Do you ever wonder how a park is designed? Ever wondered why some parks have sports fields, while others have swings, slides, and jungle gyms? Well, you would need to ask a landscape architect.

Landscape architects are people who design outdoor and public places. Landscape architects can design parks, school grounds, office parks, public centers like youth or senior centers, and other buildings and parkways. In many places, a landscape architect is involved in the planning as well. In fact, in Ontario, Canada, and Santa Barbara, California, all designs that involve public places must have a landscape architect review and approve the designs before building can begin. How do you think landscape architects use mathematics when they are working?
Problem 1  Landscaping by Linda

Linda is a landscape architect who specializes in designing backyard patio floors. She has a large collection of different square tiles that she uses to layout her patio floor designs. When she consults with a possible client, she always takes graph or grid paper to demonstrate her designs.

1. Why do you think Linda takes graph paper when she consults with possible clients?

Linda needs to make a square patio out of 169 blue square tiles. She needs to determine how many different sized square patios she can create with 169 tiles. Linda starts with the designs shown.

2. What is the area of each square patio design? Complete the table.

<table>
<thead>
<tr>
<th>Dimensions of Square Patio Design</th>
<th>Area (square units)</th>
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3. Describe the different ways you can determine the area of each square.

The area of a square is calculated by multiplying the length of the side by itself. The formula $A = s \times s$ can be written as $A = s^2$.

To calculate the **square of a number** you multiply the number by itself.
In Question 2, you calculated the area for the first three square patio designs:

\[ 1^2 = 1, \ 2^2 = 4, \ \text{and} \ 3^2 = 9. \]

The 1, 4, and 9 are called **perfect squares** because each is the square of a whole number. For instance, 9 is a perfect square because 3 is a whole number and \( 3 \times 3 = 9 \). Another way you can write this mathematical sentence is \( 3^2 = 9 \).

4. Complete the grid by continuing to create squares with side lengths of 4 through 15. Use a straightedge to connect the side lengths together, and then write the mathematical sentence using exponents to represent each perfect square.

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If you know the area of a square, you can work backwards to calculate the length of the side of the square.

For example, to determine the length of a side of a square that has an area of 81, you need to calculate what number multiplied by itself will equal 81. Since $9 \times 9 = 81$, the side length of the square is 9, and 9 is called the square root of 81.

A **square root** is one of two equal factors of a given number. Every positive number has two square roots: a positive square root and a negative square root.

For instance, 5 is the square root of 25 because $5 \times 5 = 25$. The symbol, $\sqrt{}$, is called a **radical** and it is used to indicate square roots. The **radicand** is the quantity under a radical sign.

This is read as “the square root of 25,” or as “radical 25.”

5. Write the square root for each perfect square. Use the grid you completed in Question 4.

   a. $\sqrt{1} = \underline{\hspace{1cm}}$  
   b. $\sqrt{4} = \underline{\hspace{1cm}}$  
   c. $\sqrt{9} = \underline{\hspace{1cm}}$

   d. $\sqrt{16} = \underline{\hspace{1cm}}$  
   e. $\sqrt{25} = \underline{\hspace{1cm}}$  
   f. $\sqrt{36} = \underline{\hspace{1cm}}$

   g. $\sqrt{49} = \underline{\hspace{1cm}}$  
   h. $\sqrt{64} = \underline{\hspace{1cm}}$  
   i. $\sqrt{81} = \underline{\hspace{1cm}}$

   j. $\sqrt{100} = \underline{\hspace{1cm}}$  
   k. $\sqrt{121} = \underline{\hspace{1cm}}$  
   l. $\sqrt{144} = \underline{\hspace{1cm}}$

   m. $\sqrt{169} = \underline{\hspace{1cm}}$  
   n. $\sqrt{196} = \underline{\hspace{1cm}}$  
   o. $\sqrt{225} = \underline{\hspace{1cm}}$

6. What do you think is the value of $\sqrt{0}$? Explain your reasoning.

7. What is the side length of the largest square Linda can create with 169 squares? Explain your reasoning.

8. Do you think the square root of a number will always be a whole number? If not, provide an example of a square root that is not a whole number.
The square root of most numbers is not an integer. You can estimate the square root of a number that is not a perfect square. Begin by determining the two perfect squares closest to the radicand so that one perfect square is less than the radicand, and one perfect square is greater than the radicand. Then, use trial and error to determine the best estimate for the square root of the number.

To estimate $\sqrt{10}$ to the nearest tenth, identify the closest perfect square less than 10 and the closest perfect square greater than 10.

<table>
<thead>
<tr>
<th>The closest perfect square less than 10:</th>
<th>The square root you are estimating:</th>
<th>The closest perfect square greater than 10:</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>$\sqrt{10}$</td>
<td>16</td>
</tr>
</tbody>
</table>

You know:

$\sqrt{9} = 3$  \hspace{1cm}  $\sqrt{16} = 4$

This means the estimate of $\sqrt{10}$ is between 3 and 4.

Next, choose decimals between 3 and 4, and calculate the square of each number to determine which one is the best estimate.

Consider:

$$(3.1)(3.1) = 9.61$$  
$$(3.2)(3.2) = 10.24$$

So, $\sqrt{10} = 3.1$

The symbol $\approx$ means approximately equal to.

So, the $\sqrt{10}$ is between $\sqrt{9}$ and $\sqrt{16}$. Why can't I say it's between $\sqrt{1}$ and $\sqrt{25}$?
9. Identify the two closest perfect squares, one greater than the radicand and one less than the radicand. Use the grid you completed in Question 4.
   a. \( \sqrt{8} \)
   
   b. \( \sqrt{45} \)
   
   c. \( \sqrt{70} \)
   
   d. \( \sqrt{91} \)

10. Estimate the location of each square root in Question 9 on the number line. Then, plot and label a point for your estimate.

11. Estimate each radical in Question 9 to the nearest tenth. Explain your reasoning.
   a. \( \sqrt{8} \)

   b. \( \sqrt{45} \)

   c. \( \sqrt{70} \)

   d. \( \sqrt{91} \)
12. Linda’s customer wants a square patio that has an area of 70 square meters.
   a. What is the side length of the patio? Represent the side length in radical form.

b. Estimate the side length to the nearest tenth.

### Problem 2 Making Cubes

1. Use unit cubes to build a cube with side lengths of:
   - 1 unit.
   - 2 units.
   - 3 units.

2. Complete the table.

<table>
<thead>
<tr>
<th>Dimensions of Cube</th>
<th>Number of Unit Cubes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
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<tr>
<td>$4 \times 4 \times 4$</td>
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</tbody>
</table>

The volume of a cube is calculated by multiplying the length of the cube by the width of the cube by the height of the cube; the formula $V = s \times s \times s$ can be written as $V = s^3$.

In Question 1, you calculated the volume of 3 cubes whose side lengths were the first 3 counting numbers, $1^3 = 1$, $2^3 = 8$, and $3^3 = 27$. The numbers 1, 8, and 27 are called perfect cubes. A **perfect cube** is the cube of a whole number. For example, 64 is a perfect cube since 4 is a whole number and $4 \times 4 \times 4 = 64$. To calculate the **cube of a number** you multiply the number by itself 3 times.
3. Calculate the cubes of the first 10 whole numbers.
   a. \(1^3 = \)  
   b. \(2^3 = \)  
   c. \(3^3 = \)  
   d. \(4^3 = \)  
   e. \(5^3 = \)  
   f. \(6^3 = \)  
   g. \(7^3 = \)  
   h. \(8^3 = \)  
   i. \(9^3 = \)  
   j. \(10^3 = \)  

If you know the volume of a cube, you can work backwards to calculate the side lengths of the cube. For example, to determine the side lengths of a cube that has a volume of 125, you need to calculate what number multiplied by itself 3 times will equal 125. Since \(5 \times 5 \times 5 = 125\), a side length of the cube is 5, and 5 is called the cube root of 125. A cube root is one of 3 equal factors of a number. As with the square root, the cube root also uses a radical symbol but has a 3 as an index: \(\sqrt[3]{\text{volume}}\). The index is the number placed above and to the left of the radical to indicate what root is being calculated.

4. Write the cube root for each perfect cube.
   a. \(\sqrt[3]{1} = \)  
   b. \(\sqrt[3]{8} = \)  
   c. \(\sqrt[3]{27} = \)  
   d. \(\sqrt[3]{64} = \)  
   e. \(\sqrt[3]{125} = \)  
   f. \(\sqrt[3]{216} = \)  
   g. \(\sqrt[3]{343} = \)  
   h. \(\sqrt[3]{512} = \)  
   i. \(\sqrt[3]{729} = \)  
   j. \(\sqrt[3]{1000} = \)  

5. What is the side length of the largest cube you can create with 729 cubes?

6. Will the cube root of a number always be a whole number? If not, provide an example of a cube root that is not an integer.
Most numbers do not have whole numbers for their cube root. Let’s estimate the cube root of a number using the same method used to estimate the square root of a number.

To estimate \(\sqrt[3]{33}\) to the nearest tenth, first identify the two perfect cubes closest to the radicand. One of the perfect cubes must be less than the radicand, and the other must be greater than the radicand. Then, use trial and error to determine the best estimate for the cube root.

<table>
<thead>
<tr>
<th>The closest perfect cube less than 33:</th>
<th>The cube root you are estimating:</th>
<th>The closest perfect cube greater than 33:</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
<td>(\sqrt[3]{33})</td>
<td>64</td>
</tr>
</tbody>
</table>

You know:

\[
\sqrt[3]{27} = 3 \quad \quad \quad \sqrt[3]{64} = 4
\]

This means the estimate of \(\sqrt[3]{33}\) is between 3 and 4.

Next, choose decimals between 3 and 4, and calculate the cube of each decimal to determine which one is the best estimate.

Consider:

\[
(3.2)(3.2)(3.2) = 32.768 \quad \quad \quad (3.3)(3.3)(3.3) = 35.937
\]

So, \(\sqrt[3]{33} \approx 3.2\).
7. Identify the two closest perfect cubes, one greater than the radicand and one less than the radicand. Then, estimate each cube root to the nearest tenth.

a. $\sqrt[3]{100}$

b. $\sqrt[3]{175}$

c. $\sqrt[3]{256}$

Remember, the radicand is under the $\sqrt[3]{}$.

Be prepared to share your solutions and methods.
First step: pull up next to the front car. Don’t forget to signal! Second step: turn the steering wheel toward the curb. Make sure to look in the rear-view mirror and back up slowly. Third step: don’t crunch the car next to you. Once the car is clear, turn the steering wheel the other way to make the car parallel to the curb. Make sure not to hit anything behind the car! When the car is in the space, turn off the engine. The perfect parallel park! But that’s the easy part. The hard part of parallel parking is figuring out whether the car will fit in the space in the first place. In 2009, mathematician Simon Blackburn at the University of London wrote a formula for a car company that could be used to determine exactly how much space a car needs for the perfect parallel park.

Why do you think a car company is interested in having this formula? Do you think that cars of the future will be able to park themselves? What numbers do you think should be used in the formula to make a self-parking car work?
Problem 1  Shipping Charges

1. The flat rate to ship a small box with dimensions of $8\frac{5}{8} \times 5\frac{3}{8} \times 1\frac{5}{8}$ is $4.95.

   a. What is the cost to ship 6 small boxes? Write a numeric expression to help you calculate the cost.

   b. What is the cost to ship 12 small boxes? Write a numeric expression to help you calculate the cost.

   c. What is the cost to ship 17 small boxes? Write a numeric expression to help you calculate the cost.

   d. Write a sentence to describe how you can determine the cost to ship any number of the small boxes.

2. What quantity or quantities changed in each part of Question 1?

3. What quantity or quantities remained the same in each part of Question 1.
In Question 1 there was one quantity that changed or “varied.” In mathematics, one of the most powerful concepts is to use a symbol, often a letter, to represent a quantity that varies. The use of letters or symbols, called **variables**, helps you write expressions to understand problem situations. Whenever you perform the same mathematical process over and over, you can write an **algebraic expression** to represent the situation. An **algebraic expression** is a mathematical phrase involving at least one variable, and sometimes numbers and operation symbols.

Let’s consider the situation from Question 1. To write an algebraic expression that represents the shipping charges for any number of small boxes, think about each of the number sentences you wrote in parts (a) through (c).

- The flat rate, $4.95,
- $4.95(6)$ The number of small boxes
- stayed the same.
- $4.95(12)$ changed each time.
- $4.95(17)$

To write an algebraic expression:

- Select a variable, say $n$ for “number of,” and assign it to the changing quantity.
  
  Let $n =$ the number of small boxes.
- Replace the given value of the number of small boxes in the numerical expression with the variable $n$.

$4.95(n)$ or $4.95n$

4. Use the algebraic expression, $4.95n$, to calculate the cost to ship:

a. 20 small boxes.

b. 100 boxes.

c. 252 boxes.
5. The business manager has reviewed expenses for the last three weeks. Determine the number of small boxes that were shipped given the total cost of the shipping receipt.
   a. How many small boxes were shipped if the total cost was $19.80?

   b. How many small boxes were shipped if the total cost was $69.30?

   c. How many small boxes were shipped if the total cost was $103.95?

   d. Write a sentence to describe how to calculate the number of small boxes that were shipped given the total cost of the shipping receipt.

6. What quantity or quantities changed each part of Question 5?

7. What quantity or quantities remained the same?

To help you calculate the costs of an unknown quantity, you can write an equation. An equation is a mathematical sentence that contains an equal sign. An equation can contain numbers, variables, or both in the same mathematical sentence.

8. Write an equation to describe this situation. Let \( n \) represent the number of small boxes, and let \( c \) represent the total shipping cost in dollars.
9. Myra is fulfilling orders and will need to request the correct amount of money for postage from the business manager. Determine if Myra has requested the correct amount of money to ship the small boxes for each order.
   a. Myra requests $39.60 from the business manager to ship eight small boxes. Does Myra have the correct amount of money? Explain your reasoning.
   b. Myra requests $168.30 from the business manager to ship 35 small boxes. Does Myra have the correct amount of money? Explain your reasoning.

**Problem 2  Planning an Event**

1. The Social Club is planning an event to kick off the annual charity drive. The Social Club has 15 total members. Each member is responsible for sending out an equal number of invitations for the upcoming event.
   a. How many invitations will each club member send if 75 people are invited to the event?
   b. How many invitations will each club member send if 225 people are invited to the event?
   c. How many invitations will each club member send if 375 people are invited to the event?
d. Write a sentence to describe how you can determine the number of invitations each club member will send.

e. What quantity or quantities changed? What quantity or quantities remained the same?

f. Write an algebraic expression that represents the number of invitations each club member will send. Let $p$ represent the number of people invited to the event.

2. Use your expression to determine the number of invitations that each member will send if the club invites:
   
a. 600 people.

   b. 900 people.
3. Determine the total number of people invited to the annual charity drive if each club member sends out a specific number of invitations.
   a. Each club member will send out 9 invitations. How many total people are invited?
   
   b. Each club member will send out 17 invitations. How many total people are invited?
   
   c. Each club member will send out 31 invitations. How many total people are invited?
   
   d. Write a sentence to describe how to calculate the total number of people invited to the event given the number of invitations sent out by each club member.
   
4. Write an equation to describe this situation. Let \( p \) represent the total number of people invited to the annual charity drive, and let \( n \) represent the number of invitations sent out by each club member.
Problem 3 Redeeming a Gift Card

1. Marilyn received a $25 gift card from the president of the Social Club for all of her hard work in organizing the social event. Complete the table to determine the price Marilyn will pay for each item if she uses her gift card.

<table>
<thead>
<tr>
<th>Item</th>
<th>Original Price of the Item</th>
<th>Value of Gift Card</th>
<th>Marilyn's Price After Using Her Gift Card</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shoes</td>
<td>$79.99</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concert ticket</td>
<td>$28.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Video game</td>
<td>$49.99</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. What quantity or quantities changed? What quantity or quantities remained the same?

3. Write a sentence to describe how you calculated Marilyn’s cost for each item if Marilyn uses her gift card.

4. Write an algebraic expression to determine the price Marilyn will pay for any item after using her $25 gift card. Let \( p \) represent the original price of the item.

5. Use your expression to determine Marilyn’s price if the original price is:
   a. $55.
   b. $114.
6. Complete the table to determine the original price of each item.

<table>
<thead>
<tr>
<th>Item</th>
<th>Original Price of the Item</th>
<th>Value of Gift Card</th>
<th>Marilyn's Price After Using Her Gift Card</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bike</td>
<td>$25</td>
<td>$84.00</td>
<td></td>
</tr>
<tr>
<td>Jeans</td>
<td>$25</td>
<td>$7.50</td>
<td></td>
</tr>
<tr>
<td>Fish for her aquarium</td>
<td>$25</td>
<td>$17.35</td>
<td></td>
</tr>
</tbody>
</table>

7. Write a sentence to describe how you calculated the original price of each item.

8. Write an equation to describe this problem situation. Let $p$ represent the original price of the item, and let $m$ represent the price Marilyn will pay for the item.
Talk the Talk

1. Describe an algebraic expression in your own words.

2. Name an advantage for writing an algebraic expression.

Be prepared to share your solutions and methods.
What’s My Number?

7.4
Writing Algebraic Expressions

Learning Goals
In this lesson, you will:
- Write expressions.
- Write algebraic expressions to determine values for real-world situations.
- Determine the parts of an algebraic expression.

Key Terms
- numerical coefficient
- constant
- evaluate an algebraic expression

What can you learn from a person's facial expressions? Well, some expressions are easy to identify. If someone smiles, it generally means that they are happy. A frown generally means someone is sad or disappointed. But can facial expressions help determine if someone is telling the truth? In fact, there is a science that is dedicated to studying the shifting of someone's eyes, the slight pause in a person's speech, and other facial expressions or body language to determine if someone is telling the truth or telling a lie. What other moods or feelings can you interpret through facial expressions? What can numerical expressions tell you about mathematics?
Problem 1 Writing Expressions

1. A school lunch costs $1.85. Use your calculator to determine how much money is collected for each situation. Write the numerical expression you typed in your calculator for each.
   a. Fifty-five students purchase a school lunch.

   b. One hundred twenty-three students purchase a school lunch.

   c. Two thousand thirteen students purchase a school lunch.

   d. One thousand five hundred twelve students purchase a school lunch.

2. Write a sentence to describe how you can determine the amount of money collected for any number of school lunches purchased.

3. Write an algebraic expression that represents the total amount of money collected. Let $n$ represent the number of school lunches purchased.
4. The cost to rent a skating rink is $215 for a two-hour party. The cost will be shared equally among all the people who attend the party. Use your calculator to determine how much each person will pay if:
   a. 25 people attend?

   Make sure you write the expression you typed in your calculator.

   b. 43 people attend?

   c. 81 people attend?

   d. 108 people attend?

5. Write a sentence to describe how you can determine how much each person will pay to attend the party.

6. Write an algebraic expression that represents how much each person will pay to attend the skate party. Let \( p \) represent the number of people attending.
7. Jimmy has three 300-minute international calling cards. Complete the table to determine how many minutes remain on each card after each call.

<table>
<thead>
<tr>
<th>Number of Minutes on the Calling Card</th>
<th>Duration of Call</th>
<th>Number of Minutes Remaining on Calling Card</th>
</tr>
</thead>
<tbody>
<tr>
<td>300 minutes</td>
<td>33 minutes</td>
<td></td>
</tr>
<tr>
<td>300 minutes</td>
<td>57 minutes</td>
<td></td>
</tr>
<tr>
<td>300 minutes</td>
<td>1 hour and 17 minutes</td>
<td></td>
</tr>
</tbody>
</table>

8. Write a sentence to describe how you calculated the number of minutes remaining on the calling card.

9. Write an algebraic expression to determine how many minutes remain after each call. Let \( c \) represent the duration of the call in minutes.

10. Write an algebraic expression that represents each situation.

   a. A pencil costs $0.17. How much will you spend if you buy \( p \) pencils?

   \[
   \text{Cost} = 0.17 \times p
   \]

   b. You can run a mile in 7 minutes 30 seconds. How long will it take you to run \( m \) miles assuming you run at a constant rate of speed?

   \[
   \text{Time} = \left(7 + \frac{30}{60}\right) \times m
   \]

   c. You have 4 bins. You want to have the same amount of snacks in each. If you have \( s \) snacks, how many snacks will be in each bin?

   \[
   \text{Snacks per bin} = \frac{s}{4}
   \]

   d. You have a sheet of 250 stickers that you want to share equally with your friends. If you have \( f \) friends, how many stickers will each friend receive?

   \[
   \text{Stickers per friend} = \frac{250}{f}
   \]
11. Write an algebraic expression that represents each word expression.
   a. 6 times a number, \(n\)  
   b. 5 more than \(c\)
   
   c. \(h\) minus 7  
   d. \(x\) plus 1 more
   
   e. 3 times \(y\)  
   f. \(m\) less than 9

You have written many different algebraic expressions, each requiring one of these operations:

- multiplication
- division
- addition
- subtraction

12. Consider each of the algebraic expressions shown.

   Example 1:
   \[3.45n\]

   Example 2:
   \[\frac{n}{7}\]

   Example 3:
   \[n + 12\]

   a. Write a sentence to describe each example.

   A number, or quantity, that is multiplied by a variable in an algebraic expression is called the \textbf{numerical coefficient}. If the variable does not have a coefficient, then the numerical coefficient is understood to be 1. A number, or quantity, that does not change its value is called a \textbf{constant}.

   b. State the numerical coefficients in each example.

   c. State the constants in each example.
13. Write the meaning of each algebraic expression in two ways.
   a. $6t + 3$
   b. $2 - 4s$

   c. $b - 5$
   d. $3x - y$

   e. $10 + r$
   f. $10r$

**Problem 2 Evaluations**

To **evaluate an algebraic expression** means to determine the value of the expression. When you evaluate an algebraic expression, you should substitute the given values for the variables, and then simplify the expression using the Order of Operations.

1. Write the meaning of each algebraic expression. Then, evaluate the algebraic expression for the given values.
   a. $3x - 4$, if $x = 10$
   b. $11 - s$, if $s = 2$

   c. $10 - z$, if $z = 8$
   d. $5 - \frac{y}{4}$, if $y = 2$
e. \(7 + 5a\), if \(a = 20\)
f. \(\frac{b}{4}\), if \(b = 8\)

2. Complete each table.

<table>
<thead>
<tr>
<th></th>
<th>(h)</th>
<th>(3h - 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\frac{7}{3})</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\frac{5}{6})</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>(m)</th>
<th>(1 + m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\frac{2}{3})</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.7</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>(z)</th>
<th>(\frac{2z}{3} + 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>11</td>
<td></td>
</tr>
</tbody>
</table>

Be prepared to share your solutions and methods.
The value of a baseball card in 1920 may have been a few cents. The same card today may have a value of over $10,000. A house in Ohio may have a value of $150,000. The same house on the beach in South Carolina, Georgia, or California could have a value of over $1,000,000. At one grocery store, a box of cereal may have a value of $2.38, but at another grocery store, the same box may sell for $2.69. Why do the same items sometimes have different values? How do we determine the value of things? How do we determine value in mathematics?
Problem 1  Searching for Patterns

1. Calculate the perimeter of each shape. Each square is one unit by one unit.
   a. Shape 1  Shape 2  Shape 3
      
   b. Draw and label the next three shapes following the pattern from Question 1, part (a).
      Shape 4  Shape 5  Shape 6

2. Calculate the perimeter of each shape you drew. Complete the table with your calculations.

<table>
<thead>
<tr>
<th>Shape Number</th>
<th>Perimeter (units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>
3. Calculate the perimeter of the seventh shape using the table. Explain how the table helped you determine the perimeter of the seventh shape.

4. Calculate the perimeter of the twentieth shape. Explain how you calculated your answer.

5. Write an algebraic expression that describes the relationship between the shape number and the perimeter. Define your variable.

6. If the pattern continues, what shape has a perimeter of 500 units? Explain how you determined your answer.
7. Use your table from Question 2 to plot the points on the graph.

8. Would it make sense to connect the points on this graph? Explain why or why not.

This problem situation was represented in several ways:

- a diagram of figures
- a table of values
- a verbal description
- an algebraic expression, and
- a graph.

These are often called **multiple representations** of a problem situation.
Problem 2 Using Tables of Values

1. The table shows the cost of a particular item.

<table>
<thead>
<tr>
<th>Number of items</th>
<th>Cost (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>48</td>
</tr>
<tr>
<td>16</td>
<td>96</td>
</tr>
<tr>
<td>5</td>
<td>30</td>
</tr>
</tbody>
</table>

a. Describe how the cost is related to the number of items.

b. Define a variable for the number of items.

c. Write an algebra expression to represent the cost.

d. Use the expression to calculate the cost of 12 items, and then enter the values in the table.
2. Use your table to construct a graph. Be sure to label the axes.

3. Would it make sense to connect the points on this graph? Explain why or why not.
Problem 3 Using Verbal Descriptions

1. A water tank holds 100 gallons of water. The tank is leaking at the rate of two gallons a minute. Determine how many gallons of water will be left in the tank if the leak continues for:
   a. one minute.

   b. 10 minutes.

   c. 34 minutes.

   d. 25 minutes.

2. Describe how you calculated each answer.

3. Define a variable for the quantity that changes. Then, write an algebraic expression for the amount of water in the tank.
4. How long will it take for the tank to be empty? Explain your reasoning.

5. Complete the table.

<table>
<thead>
<tr>
<th>Number of Minutes the Tank is Leaking</th>
<th>Gallons of Water Remaining in the Tank</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td></td>
</tr>
<tr>
<td>34</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
</tr>
</tbody>
</table>

6. Use your table to construct a graph. Be sure to label the axes.

7. Would it make sense to connect the points on this graph? Explain why or why not.
This graph shows the distance a car is away from home in miles versus the time in minutes.

1. Complete the table using the points from the graph.

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>Distance from home (miles)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
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<td>7</td>
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<td>8</td>
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<td>9</td>
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<tr>
<td>10</td>
<td></td>
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<td>11</td>
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<tr>
<td>12</td>
<td></td>
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<td>13</td>
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<td>17</td>
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<td>18</td>
<td></td>
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<td></td>
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<td>19</td>
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<td>20</td>
<td></td>
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<td>21</td>
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<td>22</td>
<td></td>
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<tr>
<td>23</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Define a variable for the time in minutes, and then write an expression for the distance from the home.
3. Use the expression to calculate the distance the car is from home after 20 minutes.

4. Would it make sense to connect the points on this graph? Explain why or why not.

Be prepared to share your solutions and methods.
Global positioning systems, or simply GPS, are a technology that has made people with map phobias more comfortable with directions.

Today’s GPS systems not only give directions on how to get to a location, but can also calculate the time it should take to get there. GPS also offers multiple ways of getting to the same location, and, just in case drivers want to avoid traffic, most GPS systems can determine what the traffic might be along the way. Have you ever looked up directions to get to a specific location?
Earlier, you determined that the area of a square is calculated by multiplying the length of its side by itself. You are now going to investigate this situation using a variety of methods.

1. Draw a series of 5 squares. Start with the side length of 1 unit and increase the side length by 1 unit for each new square drawn.
2. Use your drawings to answer each.
   a. Explain why the area of a square with a side length of \( s \) units will have an area of \( s^2 \) unit squares.

   b. Determine how many unit squares must be added to the first square to get the second square, added to the second square to get the third square, added to the third square to get the fourth square, etc.

   c. How many unit squares would you have to add to the fifth square to get the next square? Explain your reasoning.

   d. What number would you have to add to the 150th square to get the 151st square? Explain how you could determine this.
3. Complete the table, which represents the areas of the squares.

<table>
<thead>
<tr>
<th>Side of the Square (units)</th>
<th>Area of the Square (square units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
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<tr>
<td>5</td>
<td></td>
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<tr>
<td>6</td>
<td></td>
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<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

4. Use your table to answer each.

   a. Determine the number of square units that must be added to the area of the first square to get the area of the second square, the number of square units that are added to the second square to get the third square, etc.

   b. How many square units would you have to add to the fifth square to get the next square? Explain your reasoning.

   c. How is this number that you add each time related to the side length of the square in the first column of your table?
5. Using the information from Question 4, what number would you have to add to the 150th square to get the 151st?

6. Was the table or the drawing more useful in answering the question about the formula for the area of a square?

7. Was the table or the drawing more useful in answering the question about the number that must be added to the 150th square to get the 151st square?

8. Use your table to plot the points on the graph.

9. Connect the points with a smooth curve. Why does this make sense in this problem situation?
10. Consider a square with an area of 70 square units. Use your graph to estimate the side length of the square.

11. Estimate the side length of the square that would have an area of 45 square units.

12. Estimate the area of the square whose side length is \( 5 \frac{1}{2} \) units.

13. How did the graph help you answer these questions? Would a drawing or a table have been more useful? Explain your reasoning.

14. Define a variable for the side length of the square and write an expression for the area of a square.

15. Use this expression to calculate the areas of squares with side lengths of:
   a. 175 units.

   b. 1120 units.
16. What advantage(s), if any, does the expression provide that the drawing, table, or graph does not?

Problem 2
Comparing Strategies Using Different Representations

When you were estimating the side length of the square given the area in Problem 1 Question 10, you were also determining the square root of a number. You wrote the formula for the area of a square as $A = s^2$, where $A$ represents the area of the square and $s$ represents the side length of the square. This formula can also be written as $s = \sqrt{A}$, which can be used to determine the side length of a square if you are given the area of the square.

In Lesson 7.2, you estimated $\sqrt{70}$ using a different strategy. You first identified the two closest perfect squares, one greater than and one less than the radicand. Because $8^2 = 64$ and $9^2 = 81$, you knew that $\sqrt{70}$ was between 8 and 9. Then, using estimation and multiplication you determined a value for $\sqrt{70}$. In Question 10, you estimated $\sqrt{70}$ using the graph.

1. Compare your estimate of $\sqrt{70}$ from Lesson 7.2 to your answer from Question 10. Are they the same?

2. Estimate $\sqrt{95}$. Use the strategy from Lesson 7.2 or the graph from Question 8. Explain your choice in strategy.
3. Estimate $\sqrt{200}$. Use the strategy from Lesson 7.2 or the graph from Question 8. Explain your choice in strategy.

4. What are the advantages of using each strategy to estimate a square root?

**Talk the Talk**

Multiple representations—including drawings or diagrams, verbal descriptions, tables, and graphs—can be useful in analyzing and solving problems.

Complete the graphic organizer describing the advantages of each representation.

- Verbal Description
- Algebraic Expression
- Table
- Graph

Be prepared to share your solutions and methods.
MULTIPLE REPRESENTATIONS

VERBAL

TABLE

GRAPH

ALGEBRAIC EXPRESSION
Evaluating Numerical Expressions with Addition, Subtraction, Multiplication, and Division

Any time you use mathematical operations, they must be performed in a certain order. In general, operations are performed from left to right. Multiplication and division should be performed before addition and subtraction.

Example

Consider the numerical expression $3 + 2 \cdot 4 - 18 \div 9$. Evaluate the expression by first performing multiplication or division from left to right and then performing addition and subtraction from left to right.

$$3 + 2 \cdot 4 - 18 \div 9$$
$$= 3 + 8 - 2$$
$$= 11 - 2$$
$$= 9$$
7.1 Evaluating Numerical Expressions Involving Exponents and Parentheses

Because exponents are repeated multiplication, they are to be performed before addition and subtraction, and they are also to be performed before multiplication and division. Parentheses are used to group numbers and operations to change the normal order in which you perform operations. Expressions inside parentheses should be performed before other operations.

Example

Consider the numerical expression $5 \cdot 3^2 + 4 \div 2 \cdot (7 - 4)$. Evaluate the expression by first performing operations inside parentheses, then evaluating exponents, and finally performing multiplication, division, addition, and subtraction from left to right.

$5 \cdot 3^2 + 4 \div 2 \cdot (7 - 4)$
$= 5 \cdot 9 + 4 \div 2 \cdot 3$
$= 45 + 2 \cdot 3$
$= 45 + 6$
$= 51$

7.1 Using the Order of Operations to Simplify and Evaluate Numerical Expressions

The Order of Operations is a set of rules that ensures the same result every time an expression is evaluated.

Order of Operations Rules

1. Evaluate expressions inside parentheses or grouping symbols such as ( ) or [ ].
2. Evaluate exponents.
3. Multiply and divide from left to right.
4. Add and subtract from left to right.

Example

The Order of Operations were not followed in Solution A. The same expression has been solved correctly using the Order of Operations in Solution B.

<table>
<thead>
<tr>
<th>Solution A</th>
<th>Solution B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8 + 16 \div 4$</td>
<td>$8 + 16 \div 4$</td>
</tr>
<tr>
<td>$= 24 \div 4$</td>
<td>$= 8 + 4$</td>
</tr>
<tr>
<td>$= 6$</td>
<td>$= 12$</td>
</tr>
</tbody>
</table>
7.2 Determining the Square of a Number

Calculate the square of a number by multiplying the number by itself. A number is a perfect square if it is the square of a whole number.

Example

The number 16 is a perfect square because 4 is a whole number and $4 \times 4 = 16$. Another way to write this mathematical sentence is $4^2 = 16$.

7.2 Calculating Square Roots

A square root is one of two equal factors of a nonnegative number. Every positive number has two square roots, a positive square root (called the principal square root) and a negative square root. To determine a square root that is not a whole number, identify the two closest perfect squares, one greater than and one less than the radicand. Then, estimate the square root to the nearest tenth.

Example

Estimate $\sqrt{23}$ to the nearest tenth.

Twenty-three is between the two perfect squares 16 and 25. This means that $\sqrt{23}$ is between 4 and 5, but closer to 5.

Because $4.7^2 = 22.09$ and $4.8^2 = 23.04$, $\sqrt{23}$ is approximately 4.8.

7.2 Determining the Cube of a Number

Calculate the cube of a number by multiplying the number by itself three times. A number is a perfect cube if it is the cube of a whole number.

Example

The number 27 is a perfect cube because 3 is a whole number and $3 \times 3 \times 3 = 27$. Another way to write this mathematical sentence is $3^3 = 27$. 
7.2 Calculating the Cube Root

A cube root is one of three equal factors of a number. To determine a cube root that is not a whole number, identify the two closest perfect cubes, one greater than and one less than the radicand. Then, estimate the cube root to the nearest tenth.

Example

Estimate \( \sqrt[3]{35} \) to the nearest tenth.
Thirty-five is between the two perfect cubes 27 and 64. This means that \( \sqrt[3]{35} \) is between 3 and 4 but closer to 3.
Because \( 3.2^3 = 32.77 \) and \( 3.3^3 = 35.94 \), \( \sqrt[3]{35} \) is approximately 3.3.

7.3 Analyzing and Solving Problems

Determine the important information in the problem situation and identify the quantity that stays the same and the quantity that changes. Write a numeric expression to help solve the problem.

Example

It costs $0.35 for one color copy, so it costs $0.35(25), or $8.75, for 25 color copies and $0.35(50), or $17.50, for 50 color copies. The cost per copy remains the same, but the number of copies changes.

7.3 Writing Algebraic Expressions and Equations

Whenever you perform the same mathematical process over and over, you can write an algebraic expression to represent the situation. The use of letters, called variables, can be used to represent quantities in a problem situation that changes. To write an algebraic expression, assign a variable to the quantity that changes and determine the operation needed to solve the problem. To go further, define the solution as a variable and set it equal to the algebraic expression. An equation is a mathematical sentence that contains an equal sign and may contain numbers, variables, or both.

Example

You buy \( s \) songs online for $0.99 each.
The expression \( 0.99s \) can be used to represent this situation.
To write an equation for the problem situation, set the expression equal to the solution, or the total cost \( t \):

\[
0.99s = t.
\]
7.3 Using Algebraic Expressions

An algebraic expression can be used to calculate the solution to a problem situation. Replace the variable with a given value to create a numerical expression and solve.

Example

One party table can seat 8 guests. Use the algebraic expression \( \frac{g}{8} \), where \( g \) represents the total number of party guests, to determine the number of tables needed for a party with 112 guests. Replace the variable, \( g \), with the number of guests, 112, to create a numerical expression and solve.

\[
\frac{112}{8} = 14
\]

7.4 Writing Algebraic Expressions

Whenever you perform the same mathematical process over and over, you can write a mathematical phrase, called an algebraic expression, to represent the situation. An algebraic expression is a mathematical phrase involving at least one variable and sometimes numbers and operation symbols. Recall that a variable is a letter or symbol that is used to represent quantities.

Example

Three brothers were given \( m \) amount of money to divide equally among them. The algebraic expression that represents how much each brother will get is shown.

\[
\frac{m}{3}
\]

7.4 Describing Algebraic Expressions

A number, or quantity, that is multiplied by a variable in an algebraic expression is called the numerical coefficient. If the variable does not have a coefficient, then the coefficient is understood to be 1. A number, or quantity, that does not change its value is called a constant.

Example

\[ 35 + x \]

Thirty-five plus any number, \( x \). The numerical coefficient is 1, and the constant is 35.
Evaluating Algebraic Expressions

To evaluate an algebraic expression means to determine its value. Substitute the given values for the variables, and then simplify using the Order of Operations rules.

Example

The meaning of the algebraic expression $27 - s$ is shown. Then, the evaluation of the expression is shown when $s = 12$.

Meaning: 27 minus $s$.

Evaluate: $27 - 12 \rightarrow$ Subtract $27 - 12$ which is a difference of 15.

$27 - 12 = 15$

Multiple Representations of a Problem Situation

A problem situation can be represented in several ways including a diagram of figures, a table of values, a verbal description, an algebraic expression, and a graph.

Example

Describe the area of the shape. The problem situation is represented as:

A diagram

```
  
```

A verbal description

A row of three squares is added in each figure.

So, the area of the shape is equal to the shape number times 3.
A table of values

<table>
<thead>
<tr>
<th>Shape Number</th>
<th>Area of Shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
</tr>
</tbody>
</table>

An algebraic expression

- Let $s$ represent the shape number.
  
The area is $3s$. 

A graph

A graph with the x-axis labeled "Shape Number" ranging from 0 to 10 and the y-axis labeled "Area of Shape" ranging from 0 to 20. The graph shows several points plotted on the grid.
Using Multiple Representations to Analyze and Solve Problems

Multiple representations—including verbal descriptions, algebraic expression, tables, and graphs—can be useful in analyzing and solving problems. Verbal descriptions can help determine quantities that change and that remain the same. Tables can organize data in numerical order. Graphs provide a quick look at data and can help estimate the value of an unknown quantity. An algebraic expression allows for the exact calculation of unknown quantities.

Example

The volume of a cube can be determined using the algebraic equation $s^3$, where $s$ is the side length. For side lengths that are not whole numbers, a graph can provide a quick estimation of $s$ for a given volume.

The volume of a cube with a side length of 3.5 is about 40.

Calculate $s^3$ where $s$ is 3.5: $(3.5)^3 = 42.88$. 

\begin{tabular}{|c|c|}
\hline
Side Length & Volume \\
\hline
1 & 1 \\
2 & 8 \\
3 & 27 \\
4 & 64 \\
5 & 125 \\
6 & 216 \\
7 & 343 \\
8 & 512 \\
9 & 729 \\
10 & 1000 \\
\hline
\end{tabular}