### 3.1 Solving Equations Using One Transformation

**Objectives:**
- Learn how to solve equations systematically using addition, subtraction, and division
- Learn how to use reciprocals to solve equations

**Solving Equations:**

**Goal:** Isolate the variable on one side by using inverse operations to move everything else to the other side.

**Inverse Operations:**
- $+$, $-$ (addition & subtraction)
- $\cdot$, $\div$ (multiplication & division)
- $\sqrt{}$, $^2$ (square root & power of two)

**Transformations that Produce Equivalent Equations:**

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Original Equation</th>
<th>Equivalent Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Add the same number to both sides.</td>
<td>$x - 3 = 5$</td>
<td>$x = 8$</td>
</tr>
<tr>
<td>2. Subtract the same number from both sides.</td>
<td>$x + 6 = 10$</td>
<td>$x = 4$</td>
</tr>
<tr>
<td>3. Multiply both sides by the same nonzero number.</td>
<td>$\frac{x}{2} = 3$</td>
<td>$x = 6$</td>
</tr>
<tr>
<td>4. Divide both sides by the same nonzero number</td>
<td>$4x = 12$</td>
<td>$x = 3$</td>
</tr>
<tr>
<td>5. Interchange the two sides</td>
<td>$7 = x$</td>
<td>$x = 7$</td>
</tr>
</tbody>
</table>
3.1 Solving Equations Using One Transformation

Steps to Show when Solving Equations Using One Transformation:

1. Write Equation
2. Show inverse operation
3. Simplify
4. Give answer
5. Check your answer

Add

(Subtract

\[ x - 5 = -13 \]
\[ -8 = n + 4 \]
\[ x - 5 + 5 = -13 + 5 \]
\[ -8 - 4 = n + 4 - 4 \]
\[ x = -8 \]
\[ -12 = n \]

- or -

- or -

\[ x - 5 = -13 \]
\[ -8 = n + 4 \]
\[ +5 \]
\[ -4 \]
\[ x = -8 \]
\[ -12 = n \]

Check:

\[ -8 - 5 = -13 \]
\[ -8 = -12 + 4 \]
\[ -13 = -13 \]
\[ -8 = -8 \]

* Keep the “=” signs lined up straight down the page as you work.
3.1 Solving Equations Using One Transformation

<table>
<thead>
<tr>
<th>Divide</th>
<th>Use Reciprocal</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4x = 128)</td>
<td>(\frac{2}{5}p = 4)</td>
</tr>
<tr>
<td>(\frac{4}{4})</td>
<td>(\frac{5}{2}\frac{2}{5}p = 4\left(\frac{5}{2}\right))</td>
</tr>
<tr>
<td>(x = 32)</td>
<td>(p = 10)</td>
</tr>
</tbody>
</table>

Check:
\[4(32) \overset{?}{=} 128\]
\[128 = 128\]

Check:
\[\frac{2}{5}(10) \overset{?}{=} 4\]
\[\frac{20}{5} = 4\]
\[4 = 4\]

**Linear Equations**: \(x\) is to the first power only, not in the denominator and not an exponent.

**Properties of Equality**:

1. If \(a = b\) then \(a + c = b + c\)  
   Addition Property
2. If \(a = b\) then \(a - c = b - c\)  
   Subtraction Property
3. If \(a = b\) then \(ac = bc\)  
   Multiplication Property
4. If \(a = b\) then \(\frac{a}{c} = \frac{b}{c}\) when \(c \neq 0\)  
   Division Property
3.1 Solving Equations Using One Transformation

Proportions: \[ \frac{\text{part}}{\text{whole}} = \frac{\text{small}}{\text{large}} \]

Similar Triangles: Matching sides and angles are in proportion

Find \( x \):

\[
\frac{\text{Small } \Delta}{\text{Large } \Delta} = \frac{4}{8} = \frac{5}{x}
\]

\[ 4x = 40 \]
\[ x = 10 \]

Keep variable in numerator:

\[
\frac{\text{Large } \Delta}{\text{Small } \Delta} = \frac{x}{7} = \frac{10}{5}
\]

\[ 7 \left( \frac{x}{7} \right) = \frac{10}{5} (7) \]

\[ x = \frac{70}{5} \]

\[ x = 14 \text{ units} \]
3.2 Solving Equations Using Two or More Transformations

Objectives:

- Learn how to use two or more transformations to solve an equation

Steps to Show when Solving Equations Using Two Transformations:

1. Write Original Equation
2. Simplify both sides of the equation if needed
3. Use inverse operations to isolate the variable
4. Give answer
5. Check your answer

Example 1:

\[ 4x - 9 = 15 \]

\[
\begin{align*}
4x & = 24 \\
x & = 6
\end{align*}
\]

Check:

\[ 4(6) - 9 \overset{?}{=} 15 \]

\[ 24 - 9 \overset{?}{=} 15 \]

\[ 15 = 15 \checkmark \]

Example 2:

\[ -17 = 3 + \frac{1}{2}x \]

\[
\begin{align*}
-3 - 3 & = \frac{1}{2}x \\
-20 & = \frac{1}{2}x \\
-40 & = x
\end{align*}
\]

Check:

\[ -17 \overset{?}{=} 3 + \frac{1}{2}(-40) \]

\[ -17 \overset{?}{=} 3 - 20 \]

\[ -17 = -17 \checkmark \]
3.2 Solving Equations Using Two or More Transformations

Example 3:

\[-2(x - 3) + 5x = 36\]

use the distributive property and re-write

\[-2x + 6 + 5x = 36\]

combine like terms

\[3x + 6 = 36\]

subtract 6 from both sides

\[
\begin{align*}
3x & = 30 \\
3 & = 3 \\
x & = 10
\end{align*}
\]

divide both sides by 3

give answer

Check:

\[-2(10 - 3) + 5(10) = 36\]

\[-2(7) + 50 = 36\]

\[-14 + 50 = 36\]

\[36 = 36 \checkmark\]

Example 4:

\[125 = \frac{5}{6}(x - 18)\]

use the distributive property

\[125 = \frac{5}{6}x - 15\]

add 15 to both sides

\[+ 15 \quad + 15\]

simplify

\[140 = \frac{5}{6}x\]

multiply by the reciprocal of \(\frac{5}{6}\)

\[\left(\frac{6}{5}\right)140 = \frac{5}{6}x\left(\frac{6}{5}\right)\]

simplify and give answer

\[168 = x\]

Check:

\[125 = \frac{5}{6}(168) - 15\]

\[125 = 140 - 15\]

\[125 = 125 \checkmark\]
3.2 Solving Equations Using Two or More Transformations

Example 5:

\[ 37 = 3(2x - 4) + 5(x + 1) \] use the distributive property
\[ 37 = 6x - 12 + 5x + 5 \] combine like terms
\[ 37 = 11x - 7 \] add 7 to both sides
\[ +7 \quad +7 \]
\[ 44 = 11x \] divide both sides by 11
\[ 11 \quad 11 \]
\[ 4 = x \] give answer

Check:

\[ ? = 3(2[4] - 4) + 5(4 + 1) \] substitute 4 for \( x \)
\[ 37 = 3(8 - 4) + 5(5) \]
\[ ? = 3(4) + 25 \]
\[ ? = 12 + 25 \]
\[ ? = 37 \checkmark \]
# 3.3 Solving Equations with Variables on Both Sides

## Objectives:

- Learn how to collect variables on one side of an equation
- Learn how to use algebraic models to answer questions about real-life situations

When you have variables on both sides of an equation, move like variables to one side of the equation. Often, it’s convenient to move like variables to the left side of the equation, but it really doesn’t matter because the answer will be the same if the variables are moved to the right.

## Example 1:

<table>
<thead>
<tr>
<th>Left Side</th>
<th>Right Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>$9x - 8 = 4x + 12$</td>
<td>$9x - 8 = 4x + 12$</td>
</tr>
<tr>
<td>$\underline{-4x} -4x$</td>
<td>$\underline{-9x} -9x$</td>
</tr>
<tr>
<td>$5x - 8 = 12$</td>
<td>$-8 = -5x + 12$</td>
</tr>
<tr>
<td>$+8 +8$</td>
<td>$-12 -12$</td>
</tr>
<tr>
<td>$\underline{5x = 20}$</td>
<td>$-20 = -5x$</td>
</tr>
<tr>
<td>$5 \quad 5$</td>
<td>$-5 -5$</td>
</tr>
<tr>
<td>$x = 4$</td>
<td>$4 = x$</td>
</tr>
</tbody>
</table>

Check:

$9(4) - 8 \overset{?}{=} 4(4) + 12$

$36 - 8 \overset{?}{=} 16 + 12$

$28 = 28 \checkmark$
3.3 Solving Equations with Variables on Both Sides

Example 2:

\[
\frac{1}{3} (15x - 24) = 8 + 4(x - 3)
\]

\[
5x - 8 = 8 + 4x - 12
\]

\[
-4x = -4x
\]

\[
x - 8 = 8 - 12
\]

\[
x - 8 = -4
\]

\[
+8 = +8
\]

\[
x = 4
\]

Check:

\[
\frac{1}{3} (15[4] - 24) \overset{?}{=} 8 + 4(4 - 3)
\]

\[
\frac{1}{3} (60 - 24) \overset{?}{=} 8 + 4(1)
\]

\[
\frac{1}{3} (36) \overset{?}{=} 12
\]

\[
12 = 12 \checkmark
\]

Algebraic Models - The Practical Application

Many retail stores offer club membership promising “better deals” on items purchased by members. However, many of these memberships require the consumer to pay an up-front membership fee. Just how good the club membership is depends upon how often you use the club services.

You want to rent video games. Store #1 charges $8/game for 3 days and does not require a membership fee. Store #2 charges a $50 membership fee but only charges club members $3/game for 3 days.

To determine which store is the better choice, create a verbal model by setting both stores costs equal to each other:
3.3 Solving Equations with Variables on Both Sides

Verbal Model:

\[
\text{Store #1: } \text{rental cost/game} \cdot \# \text{ games rented} = \text{Membership fee} + \text{rental cost/game} \cdot \# \text{ games rented} \\
\text{Store #2: } \text{rental cost/game} \cdot \# \text{ games rented} = \text{store #2 membership fee} + \text{rental cost/game} \cdot \# \text{ games rented}
\]

Labels:

\( x \) = number of games rented to be even
\( \text{Store #1 rental cost} = $8/\text{game} \)
\( \text{Store #2 rental cost} = $3/\text{game} \)
\( \text{Store #2 membership fee} = $50 \)

Equation:

\[
8 \cdot x = 50 + 3 \cdot x \\
8x = 50 + 3x \\
-3x \quad -3x \\
5x = 50 \\
5 \quad 5 \\
x = 10
\]

Renting 10 games would cost the same at either store. However, if you were to rent more than 10 games, Store #2 would cost less than Store #1. Conversely, if you rented less than 10 games total, Store #1 would cost you less money.

Consumers choosing between telephone or mobile phone companies should apply the model demonstrated above to assist in their decision on whose products and services to purchase.
3.4 Linear Equations & Problem Solving

**Objective:** Learn how to use problem solving plan for problems that fit linear models

**General Plan:**
Verbal model ➔ assign lables ➔ algebraic model ➔ solve ➔ answer the question

**Example 1:**
2 joggers on a 10k course

Jogger #1 jogs 8km/hour

Jogger #2 jogs 12km/hour

If jogger #1 has a 3k head start, can the second jogger catch up before the race is over?

(Jogger #1 speed) · (time jogged) + (head start) = (Jogger #2 speed) · (time jogged)

\[ D = r \cdot t \]

Let \( t \) = time in hours

\[
8t + 3 = 12t \\
-8t \\
3 = 4t \\
\frac{3}{4} = t
\]

Will Jogger #2 catch up before the end of the race?

Jogger #2 will catch up in \( \frac{3}{4} \) of an hour
3.4 Linear Equations & Problem Solving

Does this answer make sense?

\[ D = r \cdot t \]

Distance covered by Jogger #2

Take \( \frac{3}{4} \)(12) = 9 \text{ km} \\

Distance covered by Jogger #1

\( (8t + 3) = 8\left(\frac{3}{4}\right) + 3 \)

\( 6 + 3 = 9 \text{ km} \)

**Example 2:**

Put 4 graphs on poster board 84 cm wide.
You want an 8 cm border around the end and graphs 4 cm apart. How wide should you make each of the graphs?

Draw a picture:

1. Verbal Model:

\[ 84 = \text{border} + \text{graph} \#1 + \text{gap} + \text{graph} \#2 + \text{gap} + \text{graph} \#3 + \text{gap} + \text{graph} \#4 + \text{border} \]

2. Assign Labels:

\[ x = \text{width of graph} \]

\[ \text{border} = 8 \text{ cm} \]

\[ \text{gap} = 4 \text{ cm} \]
3.4 Linear Equations & Problem Solving

3. **Algebraic Model:**

\[ 84 = 16 + 4x + 12 \]

4. **Solve:**

\[
\begin{align*}
84 &= 28 + 4x \\
-28 &= -28 \\
56 &= 4x \\
4 &= 4 \\
14 &= x
\end{align*}
\]

Each graph should be 14 cm wide
3.5 Solving Equations That Involve Decimals

Objectives:

- Learn how to find exact and approximate solutions of equations containing decimals
- Learn how to solve problems that use decimal measurements

Equations with decimal answers and decimals in the problems

How do you round off?
Allow for some error

Example 1:

\[47x - 35 = 231\]
\[+ 35 + 35\]
\[47x = 266\]
\[47 \quad 47\]
\[x = 5.66\]
3.5 Solving Equations That Involve Decimals

Example 2:
Round to the nearest 2 decimal places

\[
19.6x - 38.19 = 0.46x + 3.90
\]

\[
+ 38.19 \quad + 38.19
\]

\[
19.6x \quad = 0.46x + 42.09
\]

\[
- 0.46x \quad - 0.46x
\]

\[
19.14x \quad = 42.09
\]

\[
19.14 \quad 19.14
\]

\[
x = 2.20
\]

Clearing the decimal first:

\[
3.4 + 7.2x = 6.7x - 13.9 \quad \text{multiply by 10}
\]

\[
34 + 72x = 67x - 139
\]

\[
- 67x \quad - 67x
\]

\[
34 + 5x = -139
\]

\[
-34 \quad -34
\]

\[
5x = -173
\]

\[
5 \quad 5
\]

\[
x = -34.6
\]
3.5 Solving Equations That Involve Decimals

**Problem:**
You have $7.25 to spend on lunch. There isn’t any sales tax, but you need to pay 15% tip. What's the most expensive meal you can order?

1. **Verbal Model:**
   
   Dinner cost + tip = total cost

   Tip = 0.15 \cdot dinner cost

2. **Label:**

   d = dinner cost

   tip = 15% of dinner cost

   total cost = $7.25

3. **Algebraic Model:**

   d + 0.15d = 7.25

4. **Solve:**

   \[
   \begin{array}{c}
   1.15d = 7.25 \\
   \frac{1.15d}{1.15} = \frac{7.25}{1.15} \\
   d = 6.3043
   \end{array}
   \]

   The answer for d must be rounded to the 2nd decimal place because it represents money, which only is counted to 2 decimal places.

   The cost of dinner cannot be greater than $6.30
3.6 Literal Equations and Formulas

**Objective:**

- Learn how to solve literal equations, especially formulas, for a specified variable

**Literal equation:** an equation that uses more than one letter as a variable

Examples: \( D = r \cdot t \quad A = bh \quad I = prt \quad C = \pi d \quad \text{etc.} \)

You can solve the equation for any of the variables.

**Equation for Distance:** \( D = r \cdot t \)

To find \( r \):

\[
\frac{D}{t} = r
\]

To find \( t \):

\[
\frac{D}{r} = t
\]

**Equation for Perimeter of a Rectangle:**

If \( P = 2L + 2W \) find \( L \) of a rectangle that is \( L \) Long and \( W \) wide

\[
P = \text{add all sides}
\]

\[
\begin{align*}
P &= 2L + 2W \\
-2W &\quad -2W \\
P - 2W &= 2L \\
2 &\quad 2 \\
L &= \frac{P - 2W}{2}
\end{align*}
\]
3.6 Literal Equations and Formulas

Simple Interest:
If Interest = \(prt\) where \(p = \text{principal}, \ i = \text{interest}, \ \text{and} \ t = \text{time}\)

How long will it take to make $50 interest on $350 at 10% interest?

\[
\frac{I}{pr} = \frac{prt}{pr} = t
\]

\[
\frac{50}{(350)(0.10)} = t
\]

\[
1.429 = t
\]

1.4 year \(\approx t\) or 1 year 6 months

Equation for a line: \(y = mx + b\)

Solve for \(y\) when \(x = \frac{4y - 16}{6}\)

\[
(6) \cdot x = \frac{4y - 16}{6} \cdot (6)
\]

\[
6x = 4y - 16 + 16
\]

\[
6x + 16 = \frac{4y}{4}
\]

\[
\frac{6}{4}x + 4 = y \quad \Rightarrow \quad y = \frac{3}{2}x + 4
\]
3.7 Exploring Data: Scatter Plots

Objectives:

- How to use a coordinate plane to match points with ordered pairs of numbers
- How to use a scatter plot

**Coordinate plane:** two number lines perpendicular (⊥) to each other

- horizontal axis (x-axis)
- vertical axis (y-axis)
- origin (0, 0) where they cross
- quadrants 4 parts of the plane
- ordered pair \((x, y)\) names point on plane
3.7 Exploring Data: Scatter Plots

**Scatter plot**: used to chart data on a grid

Example 1:

Cab Fares Compares Distance in Miles to Cost

Make a table:

<table>
<thead>
<tr>
<th>(D) Distance</th>
<th>.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>(F) Fare</td>
<td>1.50</td>
<td>1.50</td>
<td>2.00</td>
<td>2.50</td>
<td>3.00</td>
<td>3.50</td>
</tr>
</tbody>
</table>

Chart:

Do you see a trend? Price goes up with distance.

Graphs indicate a relation between two variables.

Does it matter if I put Distance or Fare first?

Answer: Yes!

Why?

Answer: Something called *independent* and *dependent variables*. 
3.7 Exploring Data: Scatter Plots

**Independent and Dependent Variables:**

When graphing it is customary to display the Independent variable on the horizontal ($x$) axis, and to display the Dependent variable on the vertical ($y$) axis.

In the previous example, the fare $F$ was dependent upon the distance $D$, so when you see ordered pairs written in the form $(D, F)$ it indicates that the variable $F$ (fare) depends on $D$ (distance).

F is the dependent variable
D is the independent variable

Conversely, if the variable axis were switched so that an ordered pair would look like $(F, D)$ it indicates that the variable $D$ (distance) depends on $F$ (fare).

D is the dependent variable
F is the independent variable
3.7 Exploring Data: Scatter Plots

Name these points:

A ( , )
B ( , )
C ( , )
D ( , )

Graph:

A (-1, 0)
B (2, 1)
C (0, 4)
D (3, -2)