You can make a conjecture that the graphs of arithmetic sequences represent linear functions. A conjecture is a mathematical statement that appears to be true, but has not been formally proven. An explicit formula for an arithmetic sequence can be rewritten in function notation to prove your conjecture.

For example, to write the explicit formula $a_n = 2 + 4(n - 1)$ as a linear function in general form, first use function notation to represent $a_n$.

$$a_n = 2 + 4(n - 1)$$

$$f(n) = 2 + 4(n - 1)$$

Next, rewrite the expression $2 + 4(n - 1)$.

$$f(n) = 2 + 4n - 4$$

Distributive Property

$$= 4n + 2 - 4$$

Commutative Property

$$= 4n - 2$$

Combine like terms.

So, $a_n = 2 + 4(n - 1)$ written in function notation is $f(n) = 4n - 2$. 
One strategy to determine if a table of values represents a linear function is to examine first differences. **First differences** are the values determined by subtracting consecutive output values when the input values have an interval of 1. If the first differences of a table of values are constant, the relationship is linear.

For example, the first differences of the table shown are constant, so the relationship is linear.

The slope, $a$, of a linear function is equal to the constant difference of an arithmetic sequence. Another name for the slope of a linear function is **average rate of change**. The formula for the average rate of change is $\frac{f(t) - f(s)}{t - s}$. This represents the change in the output as the input changes from $s$ to $t$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>22</td>
</tr>
<tr>
<td>2</td>
<td>17</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>

To determine whether a table of values represents a linear function, the slope, or average rate of change, needs to be constant between all given points.

You can use substitution or a graph to determine the output for a given input of a function.

For example, consider the function $E(t) = 15t$ which models the amount of money Marilynn earns for selling $t$ T-shirts. To determine the number of shirts she needs to sell to earn $100, substitute $100$ for $E(t)$ and solve.

\[
E(t) = 15t
\]
\[
100 = 15t
\]
\[
6.67 = t
\]

Or you can determine the intersection of the graphs of the two lines represented by the equation $100 = 15x$.

Marilynn needs to sell 6.67 T-shirts to earn $100. In terms of the context, she needs to sell 7 T-shirts.
A linear function can also be referred to as a polynomial function. A polynomial is a mathematical expression involving the sum of powers in one or more variables multiplied by coefficients. The degree of a polynomial is the greatest variable exponent in the expression. The leading coefficient of a polynomial is the numeric coefficient of the term with the greatest power.

The chart shows a few examples of polynomial functions.

<table>
<thead>
<tr>
<th>Polynomial Function</th>
<th>Degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant $P(x) = 7$</td>
<td>0</td>
</tr>
<tr>
<td>Linear $P(x) = 2x - 5$</td>
<td>1</td>
</tr>
<tr>
<td>Quadratic $P(x) = 3x^2 - 2x + 4$</td>
<td>2</td>
</tr>
<tr>
<td>Cubic $P(x) = 4x^3 - 2$</td>
<td>3</td>
</tr>
</tbody>
</table>

The structure of each linear function provides important information about the graph. The general form of a linear function is $f(x) = ax + b$, where $a$ and $b$ are real numbers and $a \neq 0$. In this form, the $a$-value is the leading coefficient, which describes the steepness and direction of the line. The $b$-value describes the $y$-intercept.

When you graph a polynomial, the degree indicates the maximum number of times the graph can cross the $x$-axis. A linear function has a degree of 1, so it crosses the $x$-axis at most one time.

The factored form of a linear function is $f(x) = a(x - c)$, where $a$ and $c$ are real numbers and $a \neq 0$. When a linear function is in factored form, the value of $x$ that makes the factor $(x - c)$ equal to zero is the $x$-intercept. This value is called the zero of the function. A zero of a function is a real number that makes the value of the function equal to zero, $f(x) = 0$.

You can set the factor $(x - c)$ equal to zero to determine the point where the graph crosses the $x$-axis.

For example, the linear function $f(x) = 4x + 8$ in factored form is $f(x) = 4(x + 2)$. Set the factor $x + 2$ equal to zero and then solve for $x$ to determine the zero of the function, which is the point at which the graph of the function will cross the $x$-axis.

\[
x + 2 = 0
\]
\[
x = -2
\]
A basic function is the simplest function of its type. For example, \( f(x) = x \) is the simplest linear function. It is in the form \( f(x) = ax + b \), where \( a = 1 \) and \( b = 0 \).

For the basic function \( f(x) = x \), the transformed function \( y = f(x) + D \) affects the output values of the function. For \( D > 0 \), the graph vertically shifts up. For \( D < 0 \), the graph vertically shifts down. The amount of shift is given by \( |D| \).

For example, the function \( y = x + 3 \) translates the graph of \( y = x \) vertically up 3 units.

\[
\begin{align*}
  y &= x \\
  y &= x + 3
\end{align*}
\]
For the basic function \( f(x) = x \), the transformed function \( y = A \cdot f(x) \) affects the output values of the function. For \( |A| > 1 \), the graph vertically stretches by a factor of \( A \) units. For \( 0 < |A| < 1 \), the graph vertically compresses by a factor of \( A \) units. For \( A < 0 \), the graph reflects across the \( x \)-axis.

For example, the function \( y = 2x \) dilates the graph of \( y = x \) by a factor of 2.

When a function is both translated and vertically dilated, the resulting function can be written in the form \( A \cdot f(x) + D \), where \( D \) represents the vertical translation of \( f(x) \) and \( A \) represents the vertical dilation of \( f(x) \).

For example, the graph of \( y = 2x + 3 \) represents both a vertical translation of 3 units and vertical dilation by a factor of 2.
Functions can be represented using tables, equations, graphs, and with verbal descriptions. Features of linear functions such as \( y \)-intercepts, slopes, and independent and dependent quantities can be determined from different representations of functions.

A table can help you calculate solutions given a few specific input values. A graph can help you determine exact solutions if the graph of the function crosses the grid lines exactly. A function can be solved for any value, so any and all solutions can be determined. Technology can allow for more accuracy when using a graph to determine a solution.

For example, suppose Tyler had $100 in his car fund. He earns $7.50 per hour at his after-school job. He works 3 hours each day, including weekends. Tyler puts all of his earned money in his car fund. How many days will it take him to have enough money to buy a car that costs $3790?

A table can be used to estimate that it will take between 100 and 175 days to buy the car. A graph can be used to estimate that it will take about 160 days to buy the car. A function will give an exact solution. It will take exactly 164 days to buy a car that costs $3790.

<table>
<thead>
<tr>
<th>( t )</th>
<th>( 100 + 22.50t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>10</td>
<td>325</td>
</tr>
<tr>
<td>20</td>
<td>550</td>
</tr>
<tr>
<td>50</td>
<td>1225</td>
</tr>
<tr>
<td>100</td>
<td>2350</td>
</tr>
<tr>
<td>175</td>
<td>4037.5</td>
</tr>
</tbody>
</table>

\[
 f(t) = 100 + 22.50d \\
 3790 = 100 + 22.50d \\
 3690 = 22.50d \\
 \frac{3690}{22.50} = \frac{22.50d}{22.50} \\
 164 = t 
\]