An exponential function is a function of the form \( f(x) = ab^x \), where \( a \) and \( b \) are real numbers, and \( b \) is greater than 0 but is not equal to 1.

Geometric sequences with positive common ratios belong in the exponential function family. The common ratio of a geometric sequence is the base of an exponential function.

If a geometric sequence represents an exponential function, you can use the Product of Powers Rule and the definition of negative exponents to rewrite the explicit formula for the sequence as an exponential function.

For example, to represent \( g_n = 45 \cdot 2^{n-1} \) using function notation, first rewrite it as \( f(n) = 45 \cdot 2^{n-1} \). Next, rewrite the expression \( 45 \cdot 2^{n-1} \).

\[
\begin{align*}
  f(n) &= 45 \cdot 2^n \cdot 2^{-1} & \text{Product Rule of Powers} \\
  f(n) &= 45 \cdot 2^{1} \cdot 2^{-1} & \text{Commutative Property} \\
  f(n) &= 45 \cdot \frac{1}{2} \cdot 2^n & \text{Definition of negative exponent} \\
  f(n) &= \frac{45}{2} \cdot 2^n & \text{Multiply}
\end{align*}
\]

So, \( g_n = 45 \cdot 2^{n-1} \) written in function notation is \( f(n) = \frac{45}{2} \cdot 2^n \).

The variable \( a \) in \( f(x) = a \cdot b^x + c \) is the \( y \)-intercept, and \( b \) is the constant ratio.
An exponential function is continuous, meaning that there is a value \( f(x) \) for every real number value \( x \). If the difference in the input values is the same, an exponential function shows a constant ratio between output values, no matter how large or how small the gap between input values. A constant ratio can be used to determine output values for integer and for non-integer inputs.

An exponential function has a horizontal asymptote, which is a horizontal line that a function gets closer and closer to but never intersects.

Consider the table and graph represented by the function \( f(x) = 4^x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>( \frac{1}{16} )</td>
</tr>
<tr>
<td>-1</td>
<td>( \frac{1}{4} )</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
</tr>
</tbody>
</table>

There are no \( x \)-intercepts. The \( y \)-intercept is at (0, 1). The horizontal asymptote is \( y = 0 \). The domain is all real numbers and the graph increases over the entire domain. The range is \( y > 0 \).

A rational exponent is an exponent that is a rational number. You can write each \( n \)th root using a rational exponent. If \( n \) is an integer greater than 1, then \( \sqrt[n]{a} = \frac{1}{a^r} \).

For example, \( \sqrt[4]{b} = b^{\frac{1}{4}} \) and \( 6^{\frac{1}{5}} = \sqrt[5]{6} \).
Write expressions with rational exponents in radical form using the known properties of integer exponents. Write the power as a product using a unit fraction. Use the power of a power rule and the definition of a rational exponent to write the power as a radical.

For example, consider the expressions $8^{\frac{2}{3}}$ and $(\sqrt[7]{c})^3$.

$8^{\frac{2}{3}} = 8^{\left(\frac{1}{2}\right)^{\frac{1}{3}}} = \left(\sqrt[3]{8}\right)^2 = \left(2\right)^2$

$(\sqrt[7]{c})^3 = c^{\left(\frac{1}{7}\right)^{\frac{1}{3}}} = c^{\frac{3}{7}}$

Rewrite radical expressions with an index of 2 by extracting square roots. This is the process of removing perfect squares from under the radical symbol.

For example, consider the irrational number represented by the radical expression $\sqrt{40}$.

$\sqrt{40} = \sqrt{4 \cdot 10} = \sqrt{4} \cdot \sqrt{10} = 2\sqrt{10}$

The product $2\sqrt{10}$ is an irrational number. The product of a nonzero rational number and an irrational number is always an irrational number, but the product of two irrational numbers can be irrational or rational.

Common bases and properties of exponents are used to solve simple exponential equations.

For example, to solve the exponential equation $2187 = 3^x$, first determine the power of 3 that gives the result of 2187: $(3)(3)(3)(3)(3)(3)(3) = 2187$, or $3^7 = 2187$.

Then rewrite the equation to show common bases: $3^7 = 3^x$.

Because the expressions on both sides of the equals sign have the same base, you can set up and solve an equation using the exponents: $7 = x$. 
For the basic exponential function $f(x) = a \cdot b^x$, where $a = 1$, the transformed function can be written as $y = A \cdot b^{(x - C)} + D$.

For the basic function, the $D$-value of the transformed function $y = f(x) + D$ affects the output values of the function. For $D > 0$, the graph vertically shifts up and for $D < 0$, the graph vertically shifts down. The amount of shift is given by $|D|$.

For example, consider $d(x)$.

\[ d(x) = h(x) - 2 \]

For the basic function, the $C$-value of the transformed function $y = f(x - C)$ affects the input values of the function. The value $|C|$ describes the number of units the graph of $f(x)$ is translated right or left. If $C > 0$, the graph is translated to the right, and if $C < 0$, the graph is translated to the left.

For example, consider $k(x)$.

\[ k(x) = h(x - 2) \]
A basic function is multiplied by $-1$ to result in a reflection across the $x$-axis, or $y = 0$. The argument of a basic function is multiplied by $-1$ to result in a reflection across the $y$-axis, or $x = 0$.

For example, consider $s(x)$ and $v(x)$.

\[ s(x) = -t(x) \]
\[ v(x) = t(-x) \]

For the basic function, the $A$-value of the transformed function $y = A \cdot f(x)$ affects the output values of the function. For $|A| > 1$, the graph vertically stretches. For $0 < |A| < 1$, the graph vertically compresses. For $A = -1$, the graph is reflected across the $x$-axis.

For the basic function, the $B$-value of the transformed function $y = f \cdot (Bx)$ affects the input values of the function. For $|B| > 1$, the graph horizontally compresses. For $0 < |B| < 1$, the graph horizontally stretches. For $B = -1$, the graph is reflected across the $y$-axis.