

INTRODUCTION

A vast number of resources for teaching mathematics in the elementary and secondary grades are now available on the Internet. Many of these resources are available for free. The resources discussed in this document are web-based tools and activities for teaching and learning mathematics. Because of the vastness of the World Wide Web and the speed with which it changes, not all resources can be included in this discussion.

WEB-BASED TOOLS AND ACTIVITIES

The websites described here provide resources for web-based tools and activities that typically focus on a specific mathematical concept or skill. A wide range of programs are available through the Internet. Many are designed to help students practice a skill, such as estimation or calculation. Some are designed to promote exploration and discovery of patterns or rules. For example, a program may show a graph of a linear equation, and students can observe changes in the graph as they change the values of the variables. Other programs simulate activities such as tossing a coin to explore probability. Many of the programs are “virtual” manipulatives, or technology-based representations of physical manipulatives, such as base 10 blocks or algebra tiles.

Web-based tools and activities can be an effective adjunct to the mathematics curriculum. Many are designed to provide visual representations of math concepts. Analyzing these representations supports understanding of more abstract, symbolic concepts (Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Stylianou, 2002); the ability to translate from one form of representation to another is an important factor in problem-solving (Gagatsis & Shiakalli, 2004). When the representations are dynamic and interactive they can aid students in seeing underlying patterns and recognizing critical elements, which also is important for grasping math concepts (Ahmed, Clark-Jeavons, & Oldknow, 2004; Arcavi, 2003 ; Sloutsky & Yaras, 2000).

The use of manipulatives in mathematics has been found to support students in understanding of math concepts (Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Cass, Cates, Smith & Jackson, 2003). Although they are used primarily in the elementary grades, they can be a valuable method of introducing new concepts and linking them to prior knowledge for secondary students, particularly those who struggle with mathematics (Gagnon & Maccini, 2001). Virtual manipulatives, which mimic the attributes of concrete manipulatives, can be as effective in supporting the development of math concepts (Clements, 1996).

Many web-based activities provide opportunities for students to practice recalling basic facts or applying procedures. While a textbook can provide only a fixed set of problems, these sites allow students to solve as many problems as they need to in order to develop a skill. Often students can vary the level of the problems, and the activities typically provide feedback on the correctness of student responses. Activities such as these can help students in developing automaticity in skills (Cummings & Elkins, 1999) and in developing strategies for recalling basic facts and solving problems (Atkinson, Derry, Renkl, & Wortham, 2000; Kalyaga, Chandler, Tuovinen, & Sweller, 2001; Ward & Sweller, 1990).

WEBSITES THAT CATALOG WEB-BASED TOOLS AND ACTIVITIES

These websites include catalogs of resources available on the web that can be searched by math topic and by grade level.

Illuminations

<http://illuminations.nctm.org/>

Illuminations, developed through a partnership between the National Council of Teachers of Mathematics (NCTM) and MarcoPolo, has been designed as a companion to illuminate the NCTM standards for mathematics. It includes links to the standards and electronic examples from the NCTM website.

Illuminations includes lessons that can be sorted by standard and by grade level. The lessons have detailed descriptions and are printable. It also includes interactive tools that support exploration of math concepts. Tools are categorized by grade level.

An additional feature is *Web Resources*, a list of links to web resources that have been reviewed by an editorial board of experts. These resources can be searched by grade level and standard.

Advertising: None.

MathTools

<http://www.mathforum.org/mathtools/>

MathTools, part of the Math Forum at Drexel University, describes itself as “a community library of technology tools, lessons, activities, and support materials for teaching and learning mathematics.” A major feature is its catalog of technology resources for math on the web. The catalog includes hundreds of tools, lessons, and activities that are categorized and searchable by grade level and content. Each has been submitted by a registered user (registration is free and open to all). Each includes a description, the technology type (e.g., Java applet, Flash, WindowsCE), and ratings, reviews, and discussions from other registered users of the site. Registered users can save any activities to their own “My Math Tools” portion of the site. The activities from Arcytech, Project Interactivate, and The National Library of Virtual Manipulatives are included in the catalog.

Another feature is the *Research Area*, which monitors and summarizes research on the uses of technology in math education.

Advertising: None

WEBSITES WITH TOOLS AND ACTIVITIES THAT SUPPORT MORE THAN ONE STANDARD

These websites include several tools and/or activities that cover a variety of math topics.

AAA Math

<http://www.aaamath.com/>

This site has practice activities for many math topics for grades kindergarten through 8. For each topic, it gives an explanation and a timed practice activity. Reports of results can be viewed and printed. Some activities are available only on a CD, which must be purchased, but many are available on the website for free.

Grade Levels

- Elementary
- Middle school

Advertising: None, but they sell a CD of activities

Arcytech – Educational Java Programs

<http://arcytech.org/java/>

This website, designed and maintained by Jacobo Bulaevsky, includes interactive tools for several manipulatives commonly used in the elementary grades – Cuisenaire rods, base 10 blocks, pattern blocks, and fraction bars. Each tool has instructions and suggested lessons. Additional programs provide lessons and practice activities in telling time and using money. The site also includes Java programs for the Pythagorean Theorem, the value of pi, and exploring fractals.

Grade Levels

- Elementary
- Middle school
- High school

Advertising: None

Cyberchase for Parents and Teachers

<http://pbskids.org/cyberchase/parentsteachers/lessons.html>

This website, created by the Educational Broadcasting Corporation, is designed to accompany Cyberchase, a television series designed to teach math concepts to students ages 8 to 12 in an engaging way. The website for parents and teachers includes online games, lesson plans, and print-based activities, all of which are linked to NCTM standards. All standards are covered except Communication and Connections. A wide variety of activities and lessons are available to use as an adjunct to the curriculum. The lesson plans include a downloadable teacher's guide, and the downloadable print activities are available in both English and Spanish. The site recommends grades 3 to 5 or ages 8 to 12.

Grade Levels

- Elementary

Advertising: None

Figure This! Math Challenges for Families

<http://www.figurethis.org/index.html>

This website, developed by the National Council of Teachers of Mathematics (NCTM), is designed to assist families of middle school students in becoming engaged with and learning math. It provides math challenges with resources to support parents in exploring these challenge problems with their children. Challenges are indexed by math content. Materials (including a PowerPoint presentation and handout materials) are provided for teachers who want to introduce Figure This! to families. All challenges are available for downloading in PDF format. The challenges are available in both English and Spanish.

Grade Levels

- Middle school

Advertising: None

Graph Paper Printer

<http://webperso.easynet.fr/~philimar/graphpapeng.htm>

Graph Paper Printer, developed by Philippe Marquis, is a software application that allows the user to create and print many different kinds of graph paper. Additional options include formats such as musical notation, elementary handwriting paper, and customizable tables. Graph Paper

Printer is a shareware product. A trial version can be downloaded at no cost, and the full version is \$20.

Grade Levels

- Elementary
- Middle school
- High school

Advertising: Sells Graph Paper Printer

National Library of Virtual Manipulatives for Interactive Mathematics

<http://matti.usu.edu/nlvm/nav/index.html>

Developed at Utah State University and funded by the National Science Foundation, the National Library of Virtual Manipulatives is a library of web-based interactive virtual manipulatives and concept tutorials.

Manipulatives are sorted by grade level and math content area, and each manipulative includes instructions, suggested activities, lesson plans, and connection to relevant NCTM standards. The manipulatives include some that are commonly used in teaching (base 10 blocks, pattern blocks, and algebra blocks).

Grade Levels

- Elementary
- Middle school
- High school

Advertising: None

PlaneMath

<http://www.planemath.com/>

This project, developed by InfoUse in cooperation with NASA, grew out of the recognition that students with physical disabilities are at a disadvantage in mathematics because of the inability to manipulate objects effectively. The authors have developed activities that focus on aeronautics, an area that typically is not considered when thinking about the future careers of students with physical disabilities. The activities have been linked to NCTM standards, and they have included a matrix connecting the standards to their activities. (As this site was created before 2000, the previous standards were used.) Activities are also connected to the aeronautical concepts they teach. The activities include lesson plans with prerequisite skills and hyperlinked vocabulary, as well as suggestions for students with disabilities. Each activity includes an interview with someone with a disability who works in aeronautics.

The many lesson plans included are helpful in incorporating PlaneMath into the curriculum. Students also can use the activities independently. There is a help page for students, with frequently asked questions to get them started, and they can explore the activities on their own at home or at school. Most of the activities are text and illustrations, but two are interactive Shockwave programs.

Grade Levels

- Middle school

Advertising: None

Project Interactivate

<http://www.shodor.org/master/interactivate/>

Project Interactivate is developed and maintained by Shodor Educational Foundation, a non-profit education and research organization that focuses on developing valid models to support understanding. It includes over 100 interactive tools and activities that allow students to explore mathematics. Tools and activities are categorized by math content (number and operations, geometry and measurement, function and algebra, and data analysis and probability). All include explanations of how to use them in teaching and why the activity is useful. Many are incorporated into lesson plans that are also available, and some of these include downloadable worksheets to use in lesson. Also included is a discussion section that models how to introduce or explains different concepts to students. An online dictionary provides definitions and links to relevant discussions.

Grade Levels

- Elementary
- Middle school
- High school

Advertising: None

Talking Calculator

<http://www.premier-programming.com/calc/talkingcalc.htm>

Talking Calculator, developed by Premier Assistive Technology, is a full functioning on-screen calculator that speaks the names of the buttons and can "read" whatever it displays. It displays the student's entries and results, as well as showing the actual equation. Numbers stored in memory are displayed as well. When students are required to show their work, they can simply cut and paste the steps into a document. The website has a short video demonstrating its use; no instructions come with the free downloaded version. Although the price listed on the website is \$14.95, you can download a free full working copy from the demo page.

Grade Levels

- Elementary
- Middle school
- High school

Advertising: Sells products developed by Premier Assistive Technology

ThinkQuest Online Math Applications

http://www.thinkquest.org/library/cat_show.html?cat_id=9

ThinkQuest is an international website-building competition sponsored by the Oracle Education Foundation. Teams of students and teachers are challenged to build websites on educational topics. These websites are published in the popular ThinkQuest Library. The library contains over 40 different math websites created for the competition, categorized by topic. Several of the sites are appropriate for high school mathematics.

Grade Levels

- Elementary
- Middle school
- High school

Advertising: None

WisWeb

http://www.fi.uu.nl/wisweb/welcome_en.html

This website contains over 70 applets, categorized by topic and grade level. Many of them have been designed to complement the *Math in Context* series. Directions are limited, but each applet has background information. Also, the website includes downloadable software, most of which is for the Texas Instruments graphing calculators.

Grade Levels

- Middle school
- High school

Advertising: None

ACTIVITIES THAT ADDRESS SPECIFIC STANDARDS

Adding Integers

<http://students.washington.edu/smithr3/integers/learning.html>

About the Activities

This website is designed to help students learn about adding positive and negative integers using the concept of “zero pairs.” The site is divided into four sections:

- *Learning.* Students view examples and try out problems.
- *Apply and Practice.* Students practice what they’ve learned, using a Jeopardy format.
- *Assessment.* Students can take a self-assessment.
- *Help.* Students can access a glossary and brief explanations of the properties applied when adding integers.

How They Address Standards

Number and Operations. The website is designed to help students understand the meanings of operations using positive and negative numbers.

How to Use

The website can be used as an adjunct to classroom instruction on the topic. The author suggests that students work in pairs on the *Learning* and *Apply and Practice* sections.

Grade Level

- Middle school (the author suggests seventh grade)

Advertising: None

Algebra/Prealgebra: Using formulas and charting in spreadsheets

<http://oregonstate.edu/~niessm/>

About the Activities

This activity was presented by Maggie Niess of Oregon State University as part of a technology workshop at the 2004 NCTM conference. It is actually a series of activities that use Excel in different ways and can help students and teachers become more familiar with the many features of Excel and the ways it can be used to help develop a better understanding of algebra and prealgebra concepts.

How They Address Standards

Algebra. Students use Excel spreadsheets to help them understand patterns, relations, and functions.

How to Use

The five activities can be used both to explore algebra concepts and to learn the features of Excel. Spreadsheets to be used for the activities can be downloaded, and solutions to all

problems are included on sheets 2 and 3 of the Excel workbook. Instructions for using Excel are not included; the activities assume some working knowledge of this program.

Grade Levels

- Middle school
- High school

Advertising: None

Developing Mathematical Thinking with Effective Questioning

http://teacherline.pbs.org/teacherline/resources/questionsheet_vma.pdf

About the Activities

These pages from PBS Teacherline provide examples of questions teachers can use to support student learning and monitor progress. Questions are categorized by their purpose, and facilitate discussion of concepts rather than statement of facts.

How They Address Standards

Communication. The questions are designed to help students communicate their mathematical thinking.

How to Use

Teachers can use these questions at any point to support deeper discussions of mathematics. The questions are presented in a downloadable and printable PDF file.

Grade Levels

- Elementary
- Middle school
- High school

Advertising: None

MathVIDS

<http://coe.jmu.edu/mathvidsr/Default.htm>

Developed by David Allsopp and his associates, MathVIDS is designed to help teachers in teaching math concepts and skills to students who are struggling. It provides detailed descriptions of the research-supported instructional strategies used with their program, as well as an overview of research on math disabilities and metacognitive strategies. Lesson plans are detailed and include different levels of scaffolding. Video clips serve as models for many of the lessons.

How They Address Standards

Number and Operations. The lessons cover basic number and operations concepts and skills from kindergarten through fifth grade.

How to Use

The authors suggest that the website can be used short-term for specific lessons as well as long-term as a professional development activity.

Grade Levels

- Kindergarten
- Elementary
- Middle school

Advertising: None

Multisensory Teaching: Positive and Negative Numbers

<http://www.resourceroom.net/math/integers.asp>

About the Activities

This web page, developed by Susan Jones, contains three lesson plans and ideas for making sense out of positive and negative numbers, including links to other resources. The page also contains references and links to other resources on teaching positive and negative integers.

How They Address Standards

Number and Operations. The lessons support students in understanding the meanings of operations on positive and negative integers.

How to Use

The three lessons present an introduction to negative numbers, a lesson on adding and subtracting, and one on multiplying with negative numbers. They are designed to support understanding, and highlight places where students typically show confusion. They also draw on real-world examples for illustration.

Grade Level

- Middle school

Advertising: None

Visual Fractions

<http://www.visualfractions.com/>

About the Activities

Visual Fractions is an online tutorial offering instruction and practice in identifying, renaming, and operating on fractions (addition, subtraction, multiplication and division). Students can choose to have problems modeled with number lines or circles.

How They Address Standards

Number and Operations. The activities help students understand different ways of representing and operating on fractions.

How to Use

Students can practice identifying, renaming, and operating on fractions with the support of an interactive visual representation of their actions. They are given feedback on the correctness of their answers and can generate and print a report of their work.

Grade Level

- Elementary
- Middle school

Advertising: None

What Is the Probability of Grabbing a Green M&M?

<http://oregonstate.edu/~niessm/>

About the Activities

This activity was presented by Maggie Niess of Oregon State University as part of a technology workshop at the 2004 NCTM conference. Students use Excel spreadsheets to analyze data on the probability of selecting a green M&M from a bag. Spreadsheets with data already entered, as well as a blank form that is set up for running your own experiment, can be downloaded.

How They Address Standards

Data Analysis and Probability. Students learn to use Excel to support their understanding and to apply basic concepts of probability.

How to Use

The activity can be used to introduce Excel as a tool for organizing and analyzing data. It also provides a good basic probability activity. Spreadsheets that show data from running the experiment 2, 5, 10, and 100 times are provided, giving students an example of how experimental probability approaches theoretical probability.

Grade Levels

- Middle school

Advertising: None

Web Turtle

<http://www.sonic.net/~nbs/webturtle/>

Description

This is a web-based Turtle Graphics program that includes all the basic commands of Turtle Graphics, and several examples demonstrating how to use it.

How They Address Standards

Geometry. It can support students in analyzing characteristics needed to create two-dimensional shapes and in using visualization and spatial reasoning to predict the commands needed for drawing.

How to Use

The program includes all commands and several examples that can be modified. It can be used as an exploratory program, or students can be given specific shapes to draw and share their results. It can be used individually, by groups of students, or with one computer projection to facilitate class discussion.

Grade Levels

- Elementary
- Middle school

Advertising: None

REFERENCES

- Ahmed, A., Clark-Jeavons, A., & Oldknow, A. (2004). How can teaching aids improve the quality of mathematics education. *Educational Studies in Mathematics*, 56, 313-328.
- Arcavi, A. (2003). The role of visual representations in the learning of mathematics. *Educational Studies in Mathematics*, 52, 215-241.
- Atkinson, R.K., Derry, S.J., Renkl, A., & Wortham, D. (2000). Learning from examples: Instructional principles from the worked examples research. *Review of Educational Research*, 70(2), 181-214.
- Butler, F.M., Miller, S.P., Crehan, K., Babbitt, B., & Pierce, T. (2003). Fraction instruction for students with mathematics disabilities: Comparing two teaching sequences. *Learning Disabilities Research & Practice*, 18(2), 99-111.
- Cass, M., Cates, D., Smith, M., & Jackson, C. (2003). Effects of manipulative instruction on solving area and perimeter problems by students with learning disabilities. *Learning Disabilities Research & Practice*, 18(2), 112-120.
- Clements, D. (1996). Rethinking "concrete" manipulatives. *Teaching Exceptional Children*, 2(5), 270-279.
- Cumming, J.J., & Elkins, J. (1999). Lack of automaticity in the basic addition facts as a characteristic of arithmetic learning problems and instructional needs. *Mathematical Cognition*, 5(2), 149-180.
- Gagatsis, A., & Shiakalli, M. (2004). Ability to translate from one representation of the concept of function to another and mathematical problem solving. *Educational Psychology*, 24(5), 645-657.
- Gagnon, J.C., & Maccini, P. (2001). Preparing students with disabilities for algebra. *Teaching Exceptional Children* 34(1), 8-15.
- Kalyuga, S., Chandler, P., Tuovinen, J., & Sweller, J. (2001). When problem solving is superior to studying worked examples. *Journal of Educational Psychology*, 93(3), 579-588.
- Sloutsky, V.M., & Yarlas, A.S. (2000, August). *Problem representation in experts and novices: Part 2. Underlying processing mechanisms*. Paper presented at the 22nd Annual Conference of the Cognitive Science Society, Philadelphia, PA.
- Stylianou, D.A. (2002). On the interaction of visualization and analysis; the negotiation of a visual representation in expert problem-solving. *Journal of Mathematical Behavior*, 21, 303-317.
- Ward, M., & Sweller, J. (1990). Structuring effective worked examples. *Cognition and Instruction*, 7(1), 1-39.